Scalar and vector Fields:

A continuous function of the position of a point in a region of space is called point function. The region of space in which it specifies a physical quantity is known as a field. These fields are classified into two groups

(i) Scalar field: A scalar field is defined as that region of space, whose each point is associated with a scalar point function, i.e, a continuous function which gives the value of a physical quantity as flux, potential, temperature, etc. In a scalar field, all the points having the same scalar physical quantity are connected by the means of surfaces called equal or level surfaces.

(ii) vector field: A vector field is specified by a continuous vector point functions having magnitude and direction, both of which change from point to point, in the given region of field. The method of presentation of a vector field is called vector lines, or lines of surfaces.

Gradient of a scalar field

The gradient of a scalar point function $\phi(x,y,z)$ is defined as $\nabla\phi$ and is written as

$$\text{Grad}\,\phi = \nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi$$

$$= \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

Grad ϕ is a vector quantity.

Interpretation:

Derivative of a function of one variable tells us how fast the function varies if we move.

a small distance. It means the gradient is the rate of change of a quantity with distance. For example, temperature gradient in a metal bar is the rate of change of temperature along the bar. However, for a function of three variables, the situation is more complicated, as it depends on what direction we chose to move. For a function of $\phi(x,y,z)$

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz$$

Here $d\phi$ is a measure of change in 'ϕ' that occurs when we alter all three variables by small amounts dx, dy and dz.

$$d\phi = \vec{\nabla}\phi \cdot d\vec{l}$$

where $\vec{\nabla}\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$ is the gradient of ϕ. Gradient is a vector quantity, i.e., it has both magnitude and direction

$$d\phi = \vec{\nabla}\phi \cdot d\vec{l}$$
$$= |\vec{\nabla}\phi||dl| \cos\alpha$$

where 'α' is the angle between $\vec{\nabla}\phi$ & $d\vec{l}$. Clearly the maximum change of ϕ takes place in the direction $\alpha = 0$. It means $d\phi$ is largest when we move in the direction of $\vec{\nabla}\phi$. Or in other words $\vec{\nabla}\phi$ points in the direction of maximum increase of the function ϕ.

The gradient of a scalar function ϕ is a vector whose magnitude is equal to the maximum rate of change of ϕ with respect to the space variables and whose direction is along the change.